Chapter 6

Applications of Fuzzy Logic Control

This chapter demonstrates the usefulness and capability of the fuzzy logic control (FLC) methodology presented in Chapter 5. It is applied to a variety of real life problems: investment advisory models, pest management, inventory control models, problem analysis, and potential problem analysis.[1]

6.1 Investment Advisory Models

Financial service organizations have developed various advisory investment models for clients based on age and risk tolerance. The objective is to advice clients how to allocate portions of their investments across the three main asset types: savings, income, and growth (asset allocation).

The concepts age and risk tolerance are measured on suitable scales. Age is partitioned into three groups, for instance young ($\leq$ 30 years), middle age (between 30 and 60 years), and old ($\geq$ 60 years). The risk tolerance is partitioned on a psychometric scale from 0 to 100 into low ($\leq$ 30), moderate (between 30 and 70), and high ($\geq$ 70). A questionary filled by the client help financial experts to determine his/her risk tolerance group (low, moderate, or high). Knowing the client's age and risk tolerance group and using results from previous studies presented in tables and charts, the financial experts are in a position to advise a client how to allocate money into savings, income, and growth.

A deficiency in this model is that a person 31 years old is middle age as well as a person who is 45 years old. All ages in the interval $[31, 59]$ have the same status; they equally qualify to be middle age; there is no gradation level of belonging to the interval. The same is valid for those who are young and old. Similar difficulty arises with the notion of risk tolerance.

Classical (crisp) models of this type can be improved by using FLC methodology. This is illustrated in the following case study.

Case Study 20 *Client Asset Allocation Model*

The inputs (linguistic variables) in the fuzzy logic client asset allocation model are *age* and *risk tolerance* (*risk*). The *risk* can be estimated as in Case Study 17, Parts 1–4, Chapter 5. It is important to observe that here, in comparison to Case Study 17, there are three outputs (linguistic variables), *savings*, *income*, and *equity*. Hence this is a two-input–three-output model. Nevertheless the technique in Chapter 5 can be applied but that requires the design of three decision tables (see Notes, 2, Chapter 5).

The control objective is for any given pair (*age*, *risk*) which reflects the state of a client to find how to allocate the asset to *savings, income*, and *growth*.

Assume that the financial experts describe the two input and three output variables by the terms of triangular and trapezoidal shape as follows:

$$\begin{aligned}
Age &\triangleq \{\mathbf{Y}(young), \mathbf{MI}(middle\ age), \mathbf{OL}(old)\},\\
Risk &\triangleq \{\mathbf{L}(low), \mathbf{MO}(moderate), \mathbf{H}(high)\},\\
Saving &\triangleq \{\mathbf{L}(low), \mathbf{M}(medium), \mathbf{H}(high)\},\\
Income &\triangleq \{\mathbf{L}(low), \mathbf{M}(medium), \mathbf{H}(high)\},\\
Growth &\triangleq \{\mathbf{L}(low), \mathbf{M}(medium), \mathbf{H}(high)\}.
\end{aligned}$$

They are shown on Figs. 6.1–6.3.

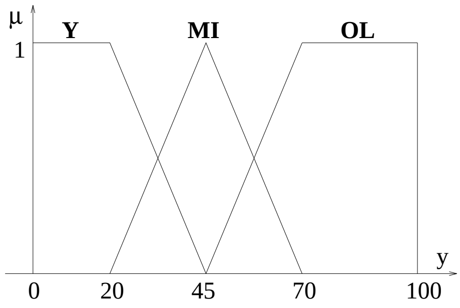

Fig. 6.1. Terms of the input *age*.

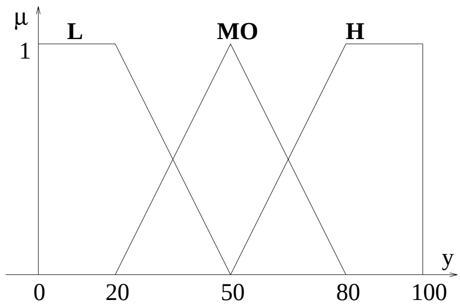

Fig. 6.2. Terms of the input *risk tolerance*.

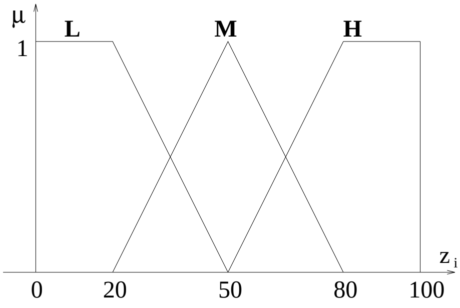

Fig. 6.3. Terms of the output variables *savings, income, growth*.

The universal sets (operating domains) of the input and output variables are $U_1 = \{x|0 \leq x \leq 100\}$ where the base variable x represents years, $U_2 = \{y|0 \leq y \leq 100\}$ with base variable y measured on a pschychometric scale, $U_3 = \{z_i|0 \leq z_i \leq 100, i = 1, 2, 3\}$ where the base variables z_i take values on scale from 0 to 100.

The terms of linguistic variables *risk, savings, income*, and *growth* are described by the same membership functions as the linguistic variables in Case Study 17 (see (5.3)). The variable *age* (Fig. 6.1) differs slightly from the other variables; the membership functions of its terms are

$$
\begin{aligned}
\mu_{\mathbf{Y}}(x) &= \begin{cases} 1 & \text{for} \quad x \le 20, \\ \frac{45-x}{25} & \text{for} \quad 20 \le x \le 45, \end{cases} \\
\mu_{\mathbf{MI}}(x) &= \begin{cases} \frac{x-20}{25} & \text{for} \quad 20 \le x \le 45, \\ \frac{70-x}{25} & \text{for} \quad 45 \le x \le 70, \end{cases} \\
\mu_{\mathbf{OL}}(x) &= \begin{cases} \frac{x-45}{25} & \text{for} \quad 45 \le x \le 70, \\ 1 & \text{for} \quad 70 \le x. \end{cases}
\end{aligned}
\tag{6.1}
$$

There are nine *if* ... *and* ... *then* rules like in Case Study 17 but each inference rule produces three (not one) conclusions, one for *savings*, one for *income*, and one for *growth*. Consequently the financial experts have to design three decision tables. Assume that these are the tables presented below.

Table 6.1. Decision table for the output *savings*.

Risk tolerance →

Age ↓		*Low*	*Moderate*	*High*
	Young	**M**	**L**	**L**
	Middle	**M**	**L**	**L**
	Old	**H**	**M**	**M**

Table 6.2. Decision table for the output *income*.

Risk tolerance →

Age ↓		*Low*	*Moderate*	*High*
	Young	**M**	**M**	**L**
	Middle	**H**	**H**	**M**
	Old	**H**	**H**	**M**

Table 6.3. Decision table for the output *growth*.

Risk tolerance →

Age ↓		*Low*	*Moderate*	*High*
	Young	**M**	**H**	**H**
	Middle	**L**	**M**	**H**
	Old	**L**	**L**	**M**

For instance the first two *if ... then* rules read:

If client's age is young and client's risk tolerance is low, then asset allocation is: medium in savings, medium in income, medium in growth.

If client's age is young and client's risk tolerance is moderate, then asset allocation is: low in savings, medium in income, high in growth.

Consider a client whose age is $x_0 = 25$ and risk tolerance level is $y_0 = 45$. Matching the readings 25 and 45 against the appropriate terms in Figs. 6.1 and 6.2 and using Eqs. (5.3) and (6.1) gives the fuzzy reading inputs

$$\mu_{\mathbf{Y}}(25) = \frac{4}{5}, \ \ \mu_{\mathbf{MI}}(25) = \frac{1}{5}, \ \ \mu_{\mathbf{L}}(45) = \frac{1}{6}, \ \ \mu_{\mathbf{MO}}(45) = \frac{5}{6}.$$

The strength of the rules calculated using (5.10) are:

$$\begin{aligned}
\alpha_{11} &= \mu_{\mathbf{Y}}(25) \wedge \mu_{\mathbf{L}}(45) = \min(\frac{4}{5}, \frac{1}{6}) = \frac{1}{6}, \\
\alpha_{12} &= \mu_{\mathbf{Y}}(25) \wedge \mu_{\mathbf{MO}}(45) = \min(\frac{4}{5}, \frac{5}{6}) = \frac{4}{5}, \\
\alpha_{21} &= \mu_{\mathbf{MI}}(25) \wedge \mu_{\mathbf{L}}(45) = \min(\frac{1}{5}, \frac{1}{6}) = \frac{1}{6}, \\
\alpha_{22} &= \mu_{\mathbf{MI}}(25) \wedge \mu_{\mathbf{MO}}(45) = \min(\frac{1}{5}, \frac{5}{6}) = \frac{1}{5}.
\end{aligned}$$

The control outputs of the rules are presented in the active cells in three decision tables (a particular case of Table 5.5).

Table 6.4. Control output *savings*.

	Low	*Moderate*
Young	$\frac{1}{6} \wedge \mu_{\mathbf{M}}(z_1)$	$\frac{4}{5} \wedge \mu_{\mathbf{L}}(z_1)$
Middle	$\frac{1}{6} \wedge \mu_{\mathbf{M}}(z_1)$	$\frac{1}{5} \wedge \mu_{\mathbf{L}}(z_1)$

Table 6.5. Control output *income*.

	Low	*Moderate*
Young	$\frac{1}{6} \wedge \mu_{\mathbf{M}}(z_2)$	$\frac{4}{5} \wedge \mu_{\mathbf{M}}(z_2)$
Middle	$\frac{1}{6} \wedge \mu_{\mathbf{H}}(z_2)$	$\frac{1}{5} \wedge \mu_{\mathbf{H}}(z_2)$

Table 6.6. Control output *growth*.

	Low	*Moderate*
Young	$\frac{1}{6} \wedge \mu_{\mathbf{M}}(z_3)$	$\frac{4}{5} \wedge \mu_{\mathbf{H}}(z_3)$
Middle	$\frac{1}{6} \wedge \mu_{\mathbf{L}}(z_3)$	$\frac{1}{5} \wedge \mu_{\mathbf{M}}(z_3)$

The outputs in the four active cells in Tables 6.4–6.6 have to be aggregated separately. The results (see Figs. 6.4–6.6) obtained by following Case Study 17 (Part 3) are:

$$\mu_{agg}(z_1) = \max\{\min(\frac{1}{6}, \mu_{\mathbf{M}}(z_1)), \min(\frac{4}{5}, \mu_{\mathbf{L}}(z_1))\};$$
$$\mu_{agg}(z_2) = \max\{\min(\frac{4}{5}, \mu_{\mathbf{M}}(z_2)), \min(\frac{1}{5}, \mu_{\mathbf{H}}(z_2))\};$$
$$\mu_{agg}(z_3) = \max\{\min(\frac{1}{5}, \mu_{\mathbf{M}}(z_3)), \min(\frac{4}{5}, \mu_{\mathbf{H}}(z_3)), \min(\frac{1}{6}, \mu_{\mathbf{L}}(z_3))\}.$$

The aggregated outputs shown on Figs. 6.4–6.6 are defuzzified by using HDM. The results are given in the same figures.

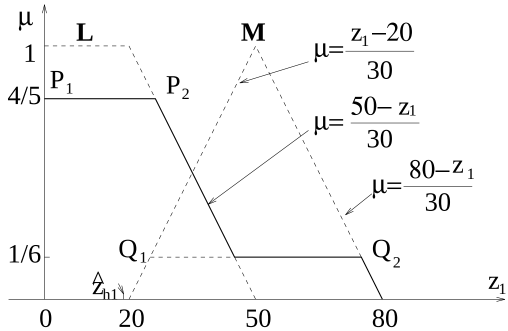

Fig. 6.4. Aggregated output *savings*. Defuzzification.

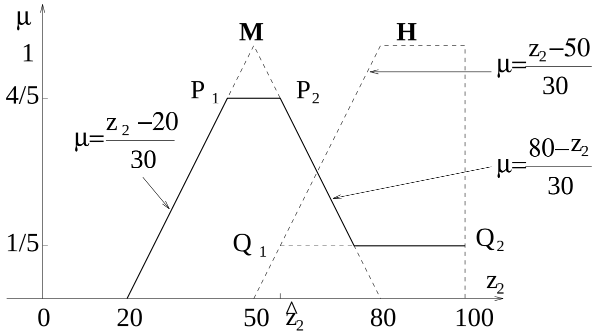

Fig. 6.5. Aggregated output *income*. Defuzzification.

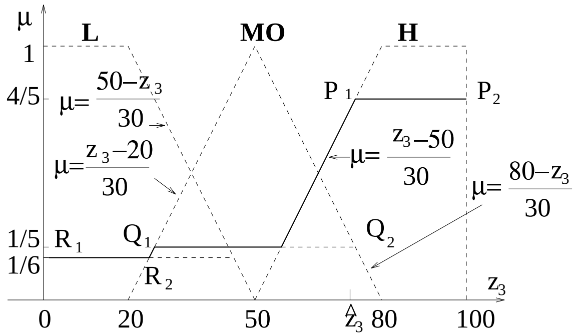

Fig. 6.6. Aggregated output *growth*. Defuzzification.

The projections of the flat segments can be easily found using their height and the relevant equations of inclined segments indicated in the figures. For instance, consider Fig. 6.4. Substituting $\frac{4}{5}$ for μ in $\mu = \frac{50-z_1}{30}$ gives the projection of P_2 to be 26. Substituting $\frac{1}{6}$ for μ in $\mu = \frac{z_1-20}{30}$ and $\mu = \frac{80-z_1}{30}$ gives the projections of Q_1 and Q_2 to be 25 and 75. Similarly one can find that the projections of P_1P_2 and Q_1Q_2 in Fig. 6.5 are the intervals [44,56] and [56, 100]. There are three flat segments P_1P_2, Q_1Q_2, and R_1R_2 in Fig. 6.6. Their projections are [74,100], [26, 74], and [0, 45].

Then using the defuzzification formula (5.19) we find

$$\hat{z}_{h1} = \frac{\frac{4}{5}\frac{0+26}{2} + \frac{1}{6}\frac{25+75}{2}}{\frac{4}{5} + \frac{1}{6}} = 19.38(saving),$$

$$\hat{z}_{h2} = \frac{\frac{4}{5}\frac{44+56}{2} + \frac{1}{5}\frac{56+100}{2}}{\frac{4}{5}+\frac{1}{5}} = 55.60(income),$$

$$\hat{z}_{h3} = \frac{\frac{4}{5}\frac{74+100}{2} + \frac{1}{5}\frac{26+74}{2} + \frac{1}{6}\frac{0+45}{2}}{\frac{4}{5}+\frac{1}{5}+\frac{1}{6}} = 71.44(growth).$$

The sum $\hat{z}_{h1} + \hat{z}_{h2} + \hat{z}_{h3} = 146.42$ represents the total asset (100%). To convert each $\hat{z}_{hi}, i = 1, 2, 3$, into percentage we use the formula

$$\frac{100\hat{z}_{hi}}{\hat{z}_{h1} + \hat{z}_{h2} + \hat{z}_{h3}} = \frac{100}{146.42}\hat{z}_{hi} = 0.68\hat{z}_{hi}, \quad i = 1, 2, 3.$$

This gives the following asset allocation of the client whose age is 25 and risk tolerance 45:

$$\begin{aligned} Savings:&\ \ 0.68(19.38)\% = 13.18\%,\\ Income:&\ \ 0.68(55.60)\% = 37.81\%,\\ Growth:&\ \ 0.68(71.44)\% = 48.58\%. \end{aligned}$$

Rounding off gives savings 13%, income 38%, and growth 49%.

These numbers can be used by financial experts as a base for making an asset allocation recommendation suitable for a person whose age is 25 and risk tolerance is 45 (on a scale from 0 to 100). □

6.2 Fuzzy Logic Control for Pest Management

There is no definite knowledge in science to tell us how to model in a unique way processes in nature, and in particular population behavior. Ecological and bio-economical systems involve various types of uncertainties and vague phenomena which makes their study extremely complicated. The better understanding of these complex systems will create conditions for better and more rational resource management and efficient control policies for restriction of undesirable growth.

In this section the fuzzy logic control (FLC) methodology is applied to population dynamics, in particular to a predator–prey system. The same methodology can be applied with some modifications to other types of interactions, for instance competition between two populations. Also it can be applied to more than two interacting populations.

Consider the prey to be a pest which serves as a host for the predator, a parasite. The pest population has size (density) x and the parasite population has size (density) y. It is assumed that the system is observable, hence the population sizes can be counted or estimated.

The predator–prey interaction takes place in a fuzzy environment due to climate conditions, diseases, harvesting, migration, interaction with other species not accounted in the system, etc. Age, sex, and genotype differences are presented in the system, and the changes in density of the populations are not only instantaneous but may depend on the past history (time-lag).

No mathematical model can describe satisfactory such a complex system. The theoretical modelers who want to derive behavior rules of general nature about the interacting populations are bound to make simplifying assumptions. They may present interesting results and elegant theorems. Unfortunately often the relation between theorems and reality is not close. Hence it is natural to look for alternative methodologies.

The control objective of the resource management is to design a growth restriction policy for the pest population (eventually extinction) by using as a control output the change (increase) in the size of the parasite; in other words to release (stock) predators in order to control pests.

We will illustrate the FLC on a case study.

Case Study 21 *Control of a Parasite–Pest System*

The number of both pests and parasites in a certain environment is assumed to vary between 0 and 16,000.

The following selections are made: inputs—*pest population size* and *parasite population size*; output—*increase of size of parasites*. They are modeled by sets of the type (5.1) each containing six terms of triangular shape. The labels of the terms are indicated in Figs. 6.7–6.9. The base variables x and y for the inputs and the base variable $\triangle y$ for the output represent numbers measuring the population sizes x and y, and the increase $\triangle y$ of the size of parasites in thousands. Equations of the segments which will be used are given in Figs. 6.7–6.8.

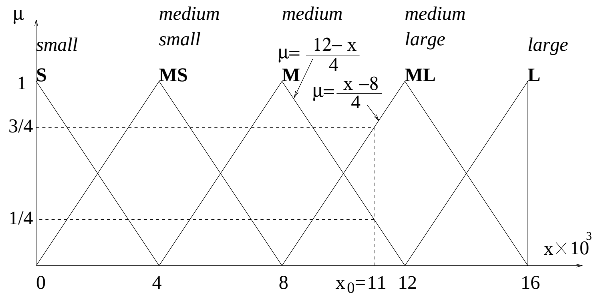

Fig. 6.7. Terms of the input *pest population size.*

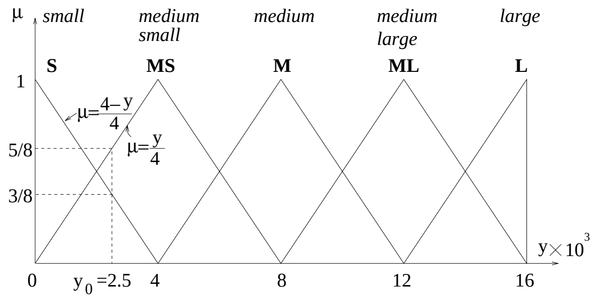

Fig. 6.8. Terms of the input *parasite population size.*

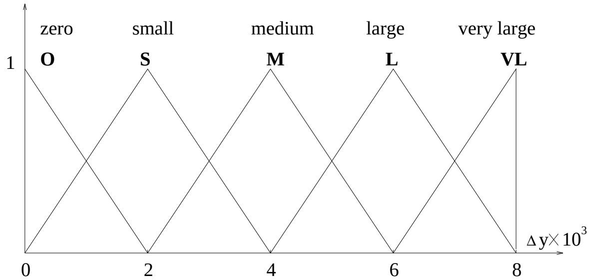

Fig. 6.9. Terms of the output *increase of parasite population size.*

The selected rules by the resource management are presented in the decision Table 6.7.

Table 6.7. *If* ... *and* ... *then* rules for parasite–pest system.

Parasite population size →

Pest population size ↓

x \ y	**S**	**MS**	**M**	**ML**	**L**
S	**0**	**0**	**0**	**0**	**0**
MS	**S**	**0**	**0**	**0**	**0**
M	**M** √	**S** √	**0**	**0**	**0**
ML	**L** √	**M** √	**S**	**0**	**0**
L	**VL**	**L**	**M**	**S**	**0**

There are 25 rules. We present only those which will be used later.

(a) *If pest population is medium and parasite population is small then exert medium increase of parasite population size.*

(b) *If pest population is medium and parasite population is medium small then exert small increase of parasite population.*

(c) *If pest population is medium large and parasite population is small then exert large increase of parasite population size.*

(d) *If pest population is medium large and parasite population is medium small then exert medium increase of parasite population size.*

Assume that at a certain time t_0 the number of pest population is estimated by resource management experts to be 11,000 or $x_0 = 11$ in thousands and the number of parasite population is estimated to be 2,500 or $y_0 = 2.5$ in thousands. The matching against appropriate terms of the input variables is shown in Figs. 6.7 and 6.8.

Using the membership function of the triangular numbers in Figs. 6.7 and 6.8 we calculate the fuzzy readings as follows. The value $x_0 = 11$ is consequently substituted for x into equations $\mu = \frac{12-x}{4}$ and $\mu = \frac{x-8}{4}$ which gives $\frac{1}{4}$ and $\frac{3}{4}$. Similarly $y_0 = 2.5$ substituted for y into equations $\mu = \frac{4-y}{4}$ and $\mu = \frac{y}{4}$ produces $\frac{3}{8}$ and $\frac{5}{8}$, correspondingly. Hence

$$\mu_{\mathbf{M}}(x_0) = \frac{1}{4}, \quad \mu_{\mathbf{ML}}(x_0) = \frac{3}{4}, \quad \mu_{\mathbf{S}}(y_0) = \frac{3}{8}, \quad \mu_{\mathbf{MS}}(y_0) = \frac{5}{8}.$$

Then the induced decision Table 5.3 reduces to the marked cells in Table 6.7 (the rest of the cells are nonactive).

The four rules to be fired are (a)–(d) induced by the marked cells in Table 6.7.

To find the levels of firing (strength of the rules) according to Section 5.5 we use formulas (5.10) which give

$$\begin{aligned}
\alpha_1 &= \mu_{\mathbf{M}}(x_0) \wedge \mu_{\mathbf{S}}(y_0) = \min(\frac{1}{4}, \frac{3}{8}) = \frac{1}{4},\\
\alpha_2 &= \mu_{\mathbf{M}}(x_0) \wedge \mu_{\mathbf{MS}}(y_0) = \min(\frac{1}{4}, \frac{5}{8}) = \frac{1}{4},\\
\alpha_3 &= \mu_{\mathbf{ML}}(x_0) \wedge \mu_{\mathbf{S}}(y_0) = \min(\frac{3}{4}, \frac{3}{8}) = \frac{3}{8},\\
\alpha_4 &= \mu_{\mathbf{ML}}(x_0) \wedge \mu_{\mathbf{MS}}(y_0) = \min(\frac{3}{4}, \frac{5}{8}) = \frac{5}{8}.
\end{aligned}$$

The control outputs of the rules (see (5.11)) are

$$\begin{aligned}
&\text{(a)} \quad \alpha_1 \wedge \mu_{\mathbf{M}}(\triangle y) = \min(\frac{1}{4}, \mu_{\mathbf{M}}(\triangle y)),\\
&\text{(b)} \quad \alpha_2 \wedge \mu_{\mathbf{S}}(\triangle y) = \min(\frac{1}{4}, \mu_{\mathbf{S}}(\triangle y)),\\
&\text{(c)} \quad \alpha_3 \wedge \mu_{\mathbf{L}}(\triangle y) = \min(\frac{3}{8}, \mu_{\mathbf{L}}(\triangle y)),\\
&\text{(d)} \quad \alpha_4 \wedge \mu_{\mathbf{M}}(\triangle y) = \min(\frac{5}{8}, \mu_{\mathbf{M}}(\triangle y)).
\end{aligned}$$

Noticing that the output of rule (a) is included into rule (d), the aggregation of the control outputs of rules (b)–(d) according to formula (5.12) produces

$$\mu_{agg}(\triangle y) = \max\{\min(\frac{1}{4}, \mu_{\mathbf{S}}(\triangle y), \min(\frac{3}{8}, \mu_{\mathbf{L}}(\triangle y)), \min(\frac{5}{8}, \mu_{\mathbf{M}}(\triangle y))\}.$$

This is a union of the three triangular fuzzy numbers **S, M, L**, presented in Fig. 6.9, sliced correspondingly with the straight lines $\mu = \frac{1}{4}, \mu = \frac{3}{8}, \mu = \frac{5}{8}$, and placed on top one other. The result is shown in Fig. 6.10 (the thick segments).

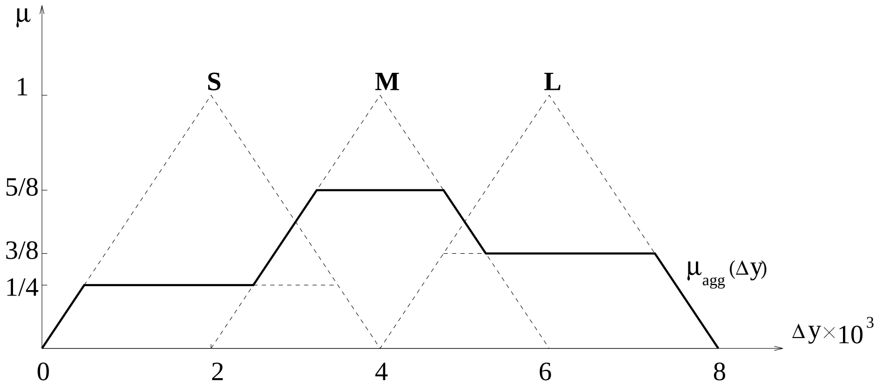

Fig. 6.10. Aggregated output for the parasite–pest system.

The mean of maximum method (MMM) is very suitable to be applied for defuzzification since precision is not important in the complex parasite–pest system under consideration. The crisp output is $\triangle\hat{y}_m = 4$ (**M** is a central triangular fuzzy number, Section 1.5.).

Hence the control action which the management should undertake is to increase the parasite population by $4 \times 10^3 = 4000$ members.

The MMM reflects only the firing of rule (d). However, the neglected rules (b) and (c) produce clipped triangulars on both sides of **M** which almost balance each another. Actually the clipped **L** (level of firing $\frac{3}{8}$) is a little bit stronger that the clipped **S** (level of firing $\frac{1}{4}$), hence MMM in this case gives a slightly conservative value which is justified from the biological point of view.

In order to make comparison, let us apply the HDM. Note that the midpoints of the flat segments of the clipped triangular numbers **S**, **M**, and **L** are 2, 4, and 6, correspondingly. Then the extended formula (5.19) (Section 5.6) gives $\triangle\hat{y}_h = 4.2$, which is close to $\triangle\hat{y}_m = 4$.

Later at a properly selected time t_1, the numbers of the prey and predator populations are to be counted or estimated. Assume they are x_1 and y_1 correspondingly. Then the whole process is to be repeated using x_1 for x_0 and y_1 for y_0. The new calculated crisp values $\triangle\hat{y}_{m1}$ will indicate what control action is needed (increase of parasite population size) to keep the pest population below 16×10^3. Again and again the same process is to be repeated.

□

6.3 Inventory Control Models

Storage cost is a major concern of production. Classical inventory models have been constructed to deal with minimizing storage cost. Their aim is to maintain enough quantities of needed parts to produce a product without incurring excessive storage cost. The product is supposed to satisfy the demand on the market. The basic inventory management problem is to decide when new parts should be ordered (order point) and in what quantities to minimize the storage cost. This is a complicated optimization problem (see for instance Fogarty and Hoffmann (1983)). Unfortunately the existing classical mathematical methods may produce a solution quite different from the real situation.

A good alternative to those methods is the FLC methodology. Its purpose is not to minimize cost directly but to maintain a proper inventory level reflecting the demand at a given time. The experience and knowledge of the managers in charge is of great importance in constructing an inventory FLC model.

The fuzzy inventory models discussed here have two input variables: *demand value* $\mathcal{D}$ for a product and *quantity-on-hand parts* (in stock) QOH needed to build the product (see Cox (1995)). There is one output variable—the *inventory action* IA which suggests reordering of parts, reducing the number of the already existing, or no action at that time.

The reduction of number of parts can be done in various ways depending on a specific situation, for instance returning parts to supplier at some nominal loss, sending parts to a sister company, etc. If this options are not available or the management decides not to use them, then the parts can be kept with anticipation demand to improve.

Inventory model 1—parts reduction possible

Following Cox (1995) we model the inputs by sets containing five terms and the output by a set containing seven terms (while Cox uses bell–shaped fuzzy numbers, we employ triangular and trapezoidal numbers):

$$Demand(\mathcal{D}) \triangleq \{\mathbf{F}, \mathbf{D}, \mathbf{S}, \mathbf{I}, \mathbf{R}\},$$

where $\mathbf{F} \triangleq falling$, $\mathbf{D} \triangleq decreased$, $\mathbf{S} \triangleq steady$, $\mathbf{I} \triangleq increased$, $\mathbf{R} \triangleq rising$;

$$Quantity\text{-}on\text{-}hand (QOH) \triangleq \{\mathbf{M}, \mathbf{L}, \mathbf{A}, \mathbf{H}, \mathbf{E}\},$$

where $\mathbf{M} \triangleq$ *minimal*, $\mathbf{L} \triangleq$ *low*, $\mathbf{A} \triangleq$ *adequate*, $\mathbf{H} \triangleq$ *high*, $\mathbf{E} \triangleq$ *excessive*;

$$Inventory\ action\ (\ IA\ ) \triangleq \{\mathbf{NL}, \mathbf{NM}, \mathbf{NS}, \mathbf{O}, \mathbf{PS}, \mathbf{PM}, \mathbf{PL}\},$$

where $\mathbf{NL} \triangleq$ *negative large*, $\mathbf{NM} \triangleq$ *negative moderate*, $\mathbf{NS} \triangleq$ *negative small*, $\mathbf{O} \triangleq$ *zero*, $\mathbf{PS} \triangleq$ *positive small*, $\mathbf{PM} \triangleq$ *positive moderate*, $\mathbf{PL} \triangleq$ *positive large*. The terms of *Inventory action* mean corresponding change to quantity-on-hand; negative stands for reduction of number of parts, positive for ordering, and zero for no action.

According to Section 5.3 the number of rules to be design is 25. They must have as a conclusion the terms of the output. Assume the management constructs the decision Table 6.8.

Table 6.8. *If* ... *and* ... *then* rules for the inventory control model.

Demand ↓ \ *Quantity–on–hand* →	*Minimal* **M**	*Low* **L**	*Adequate* **A**	*High* **H**	*Excessive* **E**
Falling **F**	**O**	**O**	**NS**	**NM**	**NL**
Decreased **D**	**PS**	**O**	**NS**	**NM**	**NM**
Steady **S**	**PM**	**PS**	**O**	**NS**	**NM**
Increased **I**	**PM**	**PM**	**PS**	**O**	**O**
Rising **R**	**PL**	**PL**	**PM**	**PS**	**O**

The rules leading to inventory action are listed below.

Rule 1: *If $\mathcal{D}$ is falling and QOH is minimal, then do nothing;*

Rule 2: *If $\mathcal{D}$ is falling and QOH is low, then do nothing;*

Rule 3: *If $\mathcal{D}$ is falling and QOH is adequate, then reduce action is negative small;*

Rule 4: *If $\mathcal{D}$ is falling and QOH is high, then reduce action is negative moderate;*

Rule 5: *If $\mathcal{D}$ is falling and QOH is excessive, then reduce action is negative large;*

Rule 6: *If $\mathcal{D}$ is decreased and QOH is minimal, then order action is positive small;*
Rule 7: *If $\mathcal{D}$ is decreased and QOH is low, then do nothing;*
Rule 8: *If $\mathcal{D}$ is decreased and QOH is adequate, then reduce action is negative small;*
Rule 9: *If $\mathcal{D}$ is decreased and QOH is high, then reduce action is negative moderate;*
Rule 10: *If $\mathcal{D}$ is decreased and QOH is excessive, then reduce action is negative large;*
Rule 11: *If $\mathcal{D}$ is steady and QOH is minimal, then order action is positive moderate;*
Rule 12: *If $\mathcal{D}$ is steady and QOH is low, then order action is positive small;*
Rule 13: *If $\mathcal{D}$ is steady and QOH is adequate, then do nothing;*
Rule 14: *If $\mathcal{D}$ is steady and QOH is high, then reduce action is negative small;*
Rule 15: *If $\mathcal{D}$ is steady and QOH is excessive, then reduce action is negative moderate;*
Rule 16: *If $\mathcal{D}$ is increased and QOH is minimal, then order action is positive moderate;*
Rule 17: *If $\mathcal{D}$ is increased and QOH is low, then order action is positive moderate;*
Rule 18: *If $\mathcal{D}$ is increased and QOH is adequate, then order action is positive small;*
Rule 19: *If $\mathcal{D}$ is increased and QOH is high, then do nothing;*
Rule 20: *If $\mathcal{D}$ is increased and QOH is excessive, then do nothing;*
Rule 21: *If $\mathcal{D}$ is rising and QOH is minimal, then order action is positive large;*
Rule 22: *If $\mathcal{D}$ is rising and QOH is low, then order action is positive large;*
Rule 23: *If $\mathcal{D}$ is rising and QOH is adequate, then order action is positive moderate;*
Rule 24: *If $\mathcal{D}$ is rising and QOH is high, then order action is positive small;*
Rule 25: *If $\mathcal{D}$ is rising and QOH is excessive, then do nothing.*

Inventory model 2—parts reduction not possible

The input variables $\mathcal{D}$ and QOH are the same introduced in Inventory model 1. Since now reduce action is not available, the output *inventory action* is partition into four terms instead of seven,

$$\textit{Inventory action (IA)} \triangleq \{\mathbf{O}, \mathbf{PS}, \mathbf{PM}, \mathbf{PL}\},$$

where $\mathbf{O}, \mathbf{PS}, \mathbf{PM}$, and **PL** have the same meaning as in Inventory model 1.

The decision table is Table 6.8 with terms **O** above the major diagonal.

Table 6.9. *If* ... *and* ... *then* rules for Inventory model 2.

Quantity-on-hand →

Demand ↓

	M	**L**	**A**	**H**	**E**
F	**O**	**O**	**O**	**O**	**O**
D	**PS**	**O**	**O**	**O**	**O**
S	**PM**	**PS**	**O**	**O**	**O**
I	**PM**	**PM**	**PS**	**O**	**O**
R	**PL**	**PL**	**PM**	**PS**	**O**

The rules producing the inventory action (the *if* ... *and* ... *then* rules) can be obtained from those for Inventory model 1 if in rules 3, 4, 5, 8, 9, 10, 14, and 15 the *then* part (conclusion) is substituted with *do nothing*; the rest of the rules remain unchanged.

The control actions discussed in this section are of qualitative nature. In order to produce a crisp action initial data (readings) are needed. This is illustrated in the following case study.

Case Study 22 *An Inventory Model with Order and Reduction Control Action.*

Assume that the input *demand* ($\mathcal{D}$) is defined on the interval $[-50, 50]$ (universal set) (Fig. 6.11) and the input *quantity-on-hand* (*QOH*) is defined on the interval [100, 200] (Fig. 6.12).

While the scale x (base variable) on which the terms of *demand* are defined is predetermined, the scale y depends on the type and number of QOH parts in a real situation.

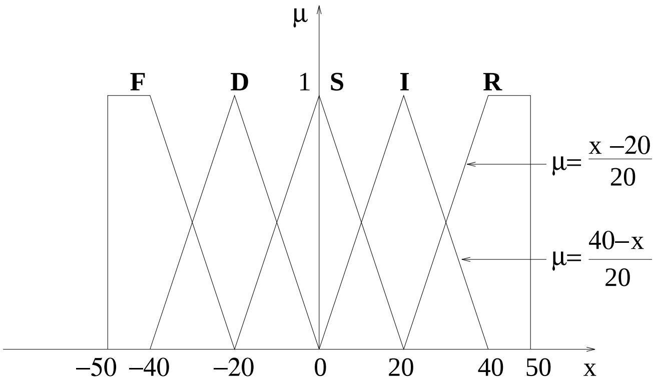

Fig. 6.11. Terms of the input variable *demand* $\mathcal{D}$.

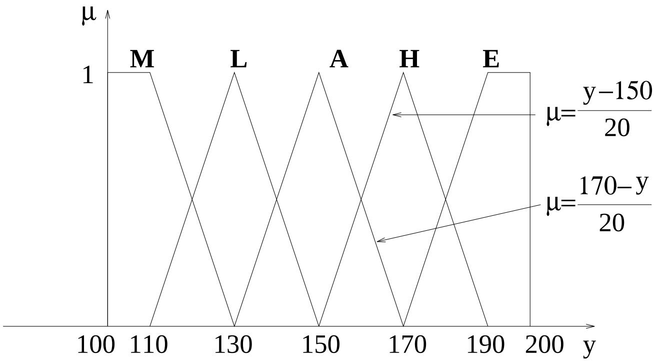

Fig. 6.12. Terms of the input variable *quantity-on-hands* (QOH).

Assume also that the output *inventory action (IA)* is defined on the interval $[-50, 50]$ (Fig. 6.13). It is a percentage scale z (base variable) whose selection depends on an estimate of the maximum number (in percentage) by which the number of inventory parts could be increased or decreased.

The terms of the inputs and the output are triangular and parts of trapezoidal numbers whose membership functions can be easily written (see Sections 1.5 and 1.6). Those to be used later (depending on the readings) are given in the figures.

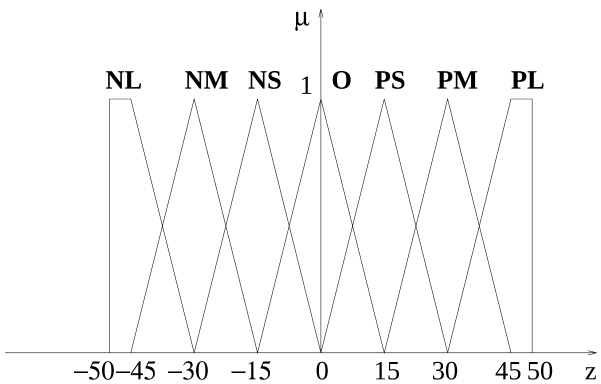

Fig. 6.13. Terms of the output variable *inventory action* (IA).

Assume that at time t_0 the demand (it has to be estimated using for instance the technique in Chapter 3, Section 4, or by other means) is $x_0 = 32$ and quantity-on-hand is $y_0 = 165$. These readings have to be matched against appropriate terms in Fig. 6.11 and Fig. 6.12. Substituting x_0 into $\mu = \frac{40-x}{20}$ and $\mu = \frac{x-20}{20}$, and y_0 into $\mu = \frac{170-y}{20}$ and $\mu = \frac{y-150}{20}$ gives

$$\mu_{\mathbf{I}}(32) = \frac{2}{5},\ \ \mu_{\mathbf{R}}(32) = \frac{3}{5},\ \ \mu_{\mathbf{A}}(165) = \frac{1}{4},\ \ \mu_{\mathbf{H}}(165) = \frac{3}{4}.$$

The induced decision Table 5.3 reduces to Table 6.10 where only the active cells are shown.

Table 6.10. Induce decision table for the inventory model.

	$\mu_{\mathbf{A}}(165) = \frac{1}{4}$	$\mu_{\mathbf{H}}(165) = \frac{3}{4}$
$\mu_{\mathbf{I}}(32) = \frac{2}{5}$	$\mu_{\mathbf{PS}}(z)$	$\mu_{\mathbf{O}}(z)$
$\mu_{\mathbf{R}}(32) = \frac{3}{5}$	$\mu_{\mathbf{PM}}(z)$	$\mu_{\mathbf{PS}}(z)$

The four rules to be fired are 18, 19, 23, 24.
The strengths of these rules are (see (5.10)):

$$\alpha_1 = \mu_{\mathbf{I}}(32) \wedge \mu_{\mathbf{A}}(165) = \min(\frac{2}{5}, \frac{1}{4}) = \frac{1}{4},$$
$$\alpha_2 = \mu_{\mathbf{I}}(32) \wedge \mu_{\mathbf{H}}(165) = \min(\frac{2}{5}, \frac{3}{4}) = \frac{2}{5},$$

$$\alpha_3 = \mu_{\mathbf{R}}(32) \wedge \mu_{\mathbf{A}}(165) = \min(\frac{3}{5}, \frac{1}{4}) = \frac{1}{4},$$
$$\alpha_4 = \mu_{\mathbf{R}}(32) \wedge \mu_{\mathbf{H}}(165) = \min(\frac{3}{5}, \frac{3}{4}) = \frac{3}{5}.$$

The control outputs (CO) of the rules are (see (5.11)):

CO of rule 18: $\alpha_1 \wedge \mu_{\mathbf{PS}}(z) = \min(\frac{1}{4}, \mu_{\mathbf{PS}}(z))$,
CO of rule 19: $\alpha_2 \wedge \mu_{\mathbf{O}}(z) = \min(\frac{2}{5}, \mu_{\mathbf{O}}(z))$,
CO of rule 23: $\alpha_3 \wedge \mu_{\mathbf{PM}}(z) = \min(\frac{1}{4}, \mu_{\mathbf{PM}}(z))$,
CO of rule 24: $\alpha_4 \wedge \mu_{\mathbf{PS}}(z) = \min(\frac{3}{5}, \mu_{\mathbf{PS}}(z))$.

The output of the rule 18 is included into that of rule 24. Hence the aggregation of the control outputs (see (5.12)) gives (Fig. 6.14):

$$\mu_{agg}(z) = \max\{\min(\frac{2}{5}, \mu_{\mathbf{O}}(z)), \min(\frac{1}{4}, \mu_{\mathbf{PM}}(z)), \min(\frac{3}{5}, \mu_{\mathbf{PS}}(z))\}.$$

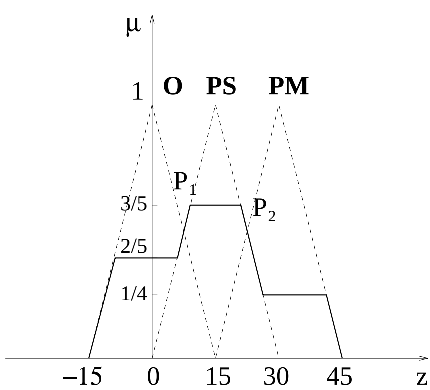

Fig. 6.14. Aggregated output for the inventory model. Defuzzification.

Similar to Case Study 21 (see Fig. 6.10), we can use for defuzzification MMM which gives $\hat{z}_m = 15$ (**PS** is triangular number in central form). Since rule 19 has level of firing $\frac{2}{5}$ which is stronger than $\frac{1}{4}$, that of the rule 23, $\hat{z}_m = 15$ is a little bit optimistic value meaning that ordering of parts is not on the conservative side. Of course the HDM, which will produce a smaller value than 15, could be easily applied (see Case Studies 20 and 21).

Now we have to translate $\hat{z}_m = 15$ (in percentage) into a corresponding inventory action. If the QOH at the time t_0 of the study ($x_0 = 32, y_0 = 165$) denoted $(QOH)_{current}$ is considered as unit 1 (or 100%), then it has to be increased by 15 %. This gives $1 + \frac{15}{100} = 1.15$ called *adjustment factor* (AF). The control action leads to a new QOH denoted $(QOH)_{new}$ which is $(QOH)_{current}$ multiplied by (AF), i.e. $165 \times 1.15 = 188.75 \approx 199$. The difference $199 - 165 = 34$ suggests that 34 new parts are to be ordered.

The following general formula can be used:

$$(QOH)_{new} = (QOH)_{current} \times AF,$$

where

$$AF = 1 + \frac{\hat{z}}{100};$$

$\hat{z}$ is a defuzzified value obtained by one of the available methods.

If $\hat{z} > 0$ like in the case discussed, the control action is ordering of new parts; if $\hat{z} < 0$, the control action is reduction.

□

6.4 Problem Analysis

Problem analysis or *deviation performance analysis* deals with problems created when there are undesirable deviations from some expected standard performance. The cause of such deviations is an unplanned and unanticipated change (see Kepner and Tregoe (1965) and Simon (1960)).

The manager or a managerial body in charge of certain areas of operation must recognize an undesirable deviation if such has developed or occurred. Also several deviations may occur concurrently. The manager must find what is wrong and what is the cause for it in order to do the necessary correction. A good knowledge of the expected performance standards in each area of operation will help the manager to identify deviations from such performance. Some deviations are permissible within certain limits established by the manager or a governing body. They have to be watched; no correction at that time is needed.

Once the manager has made sure that the deviations are identified, they have to be ranked according to their importance.

Kepner and Tregoe (1965) who contributed to classical problem analysis suggest that several important questions have to be addressed by the manager:

(1) How urgent is the deviation?
(2) How serious is the deviation?
(3) What is the deviation growth potential?
(4) What is the priority of the deviation?

The answer to these questions requires experience and skills from the manager. Valuable instructions and examples are provided by Kepner and Tregoe (1965).

Our approach in dealing with the above questions is different. We use the tools of fuzzy logic control (FLC) to quantify more realistically the classical problem analysis and arrive to conclusion.

Urgent, serious, and *growth potential* are considered here as linguistic variables; they are the inputs. The output variable is *priority of deviation.* Since high precision is not needed, we model each variable by three terms (using triangular and trapezoidal numbers):

$$\begin{aligned}
&\mathit{Urgent}(U) \triangleq \{\mathbf{N}, \mathbf{S}, \mathbf{V}\},\\
&\mathit{Serious}(S) \triangleq \{\mathbf{N}, \mathbf{S}, \mathbf{V}\},\\
&\mathit{Growth\ potential}(GP) \triangleq \{\mathbf{L}, \mathbf{M}, \mathbf{H}\},\\
&\mathit{Priority\ of\ deviation}(POD) \triangleq \{\mathbf{L}, \mathbf{M}, \mathbf{H}\},
\end{aligned}$$

where $\mathbf{N} \triangleq$ *not*, $\mathbf{S} \triangleq$ *somewhat*, $\mathbf{V} \triangleq$ *very*, $\mathbf{H} \triangleq$ *high*, $\mathbf{L} \triangleq$ *low*, $\mathbf{M} \triangleq$ *medium*.

Since we are dealing with three inputs according to Chapter 5 (Notes,2) we have to design $3 \times 3 \times 3 = 27$ rules of the type *if ... and ... and ... then.* For instance, *if deviation* (D) *is somewhat urgent and D is very serious and D growth potential is medium then priority of deviation is high.*

From these rules eight have to be fired hence the aggregated conclusion will consists of eighth (or less) superimposed clipped fuzzy numbers. This can be done but is complicated.

In order to simplify the control procedure we consider as in Chapter 5, Section 5.9, the input variables to be independent of each other meaning that the rules will be of the type *if ... then* without using *and* (*precondition*) part. This approach reduces the number of rules from 27 to 9. They are listed below in three groups concerning *urgent* (U), *serious* (S), and *growth potential* (GP); in each group there is one input and one output.

$$\left.\begin{array}{ll}\text{Rule 1:} & \textit{If D is NU then POD is L},\\ \text{Rule 2:} & \textit{If D is SU then POD is M},\\ \text{Rule 3:} & \textit{If D is VU then POD is H},\end{array}\right\} \tag{6.2}$$

$$\left.\begin{array}{ll}\text{Rule 4:} & \textit{If D is NS then POD is L},\\ \text{Rule 5:} & \textit{If D is SS then POD is M},\\ \text{Rule 6:} & \textit{If D is VS then POD is H},\end{array}\right\} \tag{6.3}$$

$$\left.\begin{array}{ll}\text{Rule 7:} & \textit{If D is with LGP then POD is L},\\ \text{Rule 8:} & \textit{If D is with MGP then POD is M},\\ \text{Rule 9:} & \textit{If D is with HGP then POD is H}.\end{array}\right\} \tag{6.4}$$

For instance, the first rule reads: *if deviation is not urgent then priority of deviation is low.*

The FLC is applied separately for each group of rules and the obtained conclusions are aggregated. In practice this means that we have to apply the simplified procedure in Section 5.9 three times for one-input–one-output control model and then to aggregate the three outputs.

Details are presented in the following case study.

Case Study 23 *Fuzzy Logic Control for Problem Analysis*

Let us assume that the three input variables and the output variable are defined on a psychometric scale [0, 100] as shown in Figs. 6.15–6.18.

Assume that the manager detects a deviation performance and gives the assessments (readings) $x_0 = 40, y_0 = 20, z_0 = 75$ of the base variables x, y, and z measuring how urgent is the deviation, how serious is it, and what is its growth potential on the scale [0, 100].

The fuzzy reading inputs generated by x_0, y_0, and z_0 are shown in Figs. 6.15–6.17. They are actually the strength of the rules (the levels of firing).

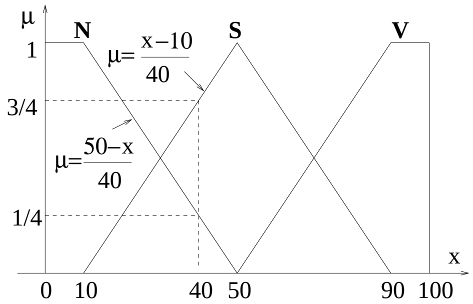

Fig. 6.15. Terms of the input variable *urgent*.

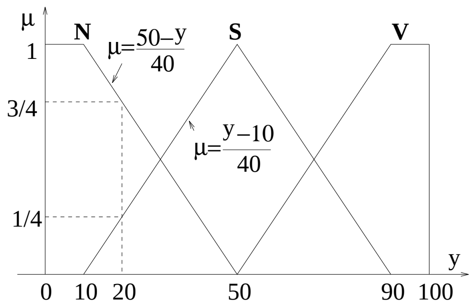

Fig. 6.16. Terms of the input variable *serious*.

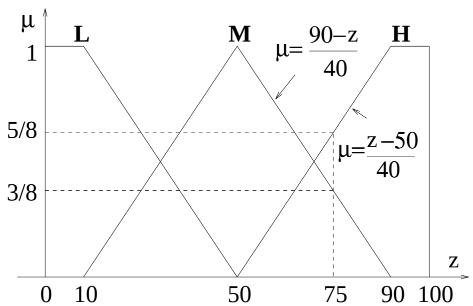

Fig. 6.17. Terms of the output variable *growth potential*.

Now the technique in Case Study 18 has to be applied three times since the three inputs U, S, and GP are considered as independent which is reflected in the three groups of rules (6.1)–(6.3). For each group the FLC requires that two rules are to be fired at specified levels. When combined they produce three independent control outputs $\mu_x(v), \mu_y(v)$, and $\mu_z(v)$ whose aggregation will give the membership function $\mu_{agg}(v)$ of the final conclusion concerning priority of deviation (POD).

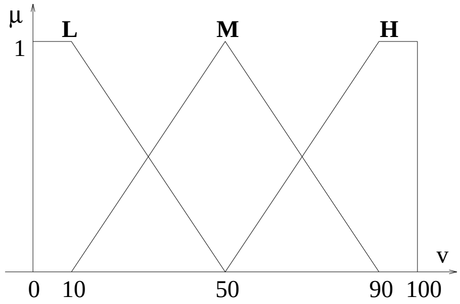

Fig. 6.18. Terms of the output variable *priority of deviation.*

The procedure is performed in Fig. 6.19. Only the relevant terms are presented.

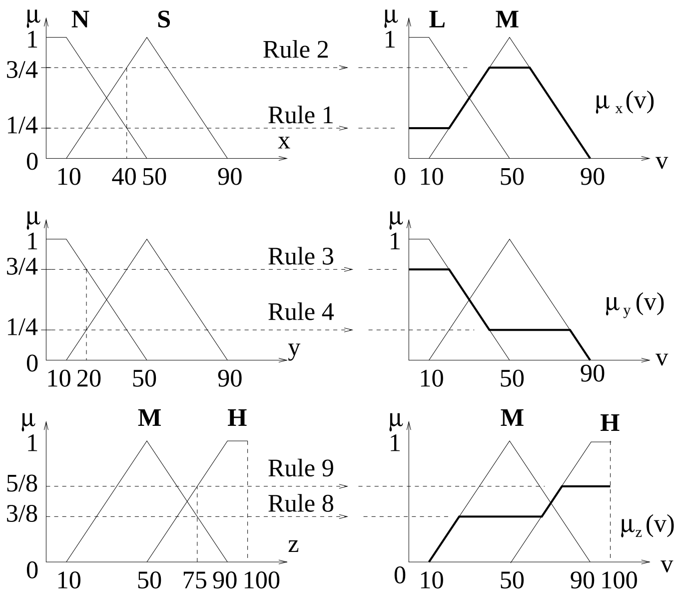

Fig. 6.19. Firing of rules for three independent inputs.

The aggregation of $\mu_x(v), \mu_y(v)$, and $\mu_z(v)$ using operation max gives the output

$$\mu_{agg}(v) = \max(\mu_x(v), \mu_y(v), \mu_z(v))$$

geometrically presented in Fig. 6.20. It is obtained by superimposing $\mu_x(v), \mu_y(v)$, and $\mu_z(v)$ a top one other (see Section 5.5).

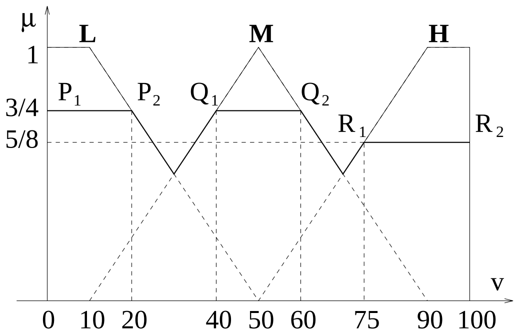

Fig. 6.20. Aggregation of the independent inputs. Defuzzification.

To defuzzify $\mu_{agg}(v)$ we use the HDM. Since the projections of the flat segments P_1P_2, Q_1Q_2, and R_1R_2 are [0,20], [40, 60], and [75, 100], the extended formula (5.19) gives

$$\hat{v}_h = \frac{\frac{3}{4}\frac{0+20}{2} + \frac{3}{4}\frac{40+60}{2} + \frac{5}{8}\frac{75+100}{2}}{\frac{3}{4} + \frac{3}{4} + \frac{5}{8}} = 46.91 \approx 47.$$

The interpretation is that the priority of deviation is almost medium; on a scale from 0 to 100 it is ranked 47. The manager will act accordingly.

□

6.5 Potential Problem Analysis

This section is closely connected to Section 6.4—Problem Analysis.

The aim of potential problem analysis is to prevent occurrence of possible problems (in the sense of undesirable deviations from certain expected performance). The bottom line is to minimize the consequences of potential problems if they do occur (see Kepner and Tregoe (1965)).

Here we use FLC methodology to model some aspects of classical problem analysis considered by Kepner and Tregoe (1965).[2]

A manager in charge of a project may find several potential problems with various degrees of risk for the project. The manager has to concentrate to those that are more dangerous on the project. The following questions are important and deserve consideration:

(1) How serious will be for the project if a potential problem (deviation) occurs?

(2) How possible is that a potential problem might occur?

(3) In what degree (magnitude) a potential problem might happen?

(4) Which are the potential problems that require attention or response?

Serious (concerning consequence of occurence of potential problem), *possible* (concerning occurence of potential problem), and *degree* (extent, magnitude, concerning partial occurence of a potential problem) are inputs; *response* is the output. They are described by fuzzy sets containing three terms.

$$\begin{aligned}
&\textit{Serious (S)} \triangleq \{\mathbf{A}, \mathbf{HU}, \mathbf{F}\},\\
&\textit{Possible (P)} \triangleq \{\mathbf{N}, \mathbf{S}, \mathbf{V}\},\\
&\textit{Degree (D)} \triangleq \{\mathbf{L}, \mathbf{M}, \mathbf{H}\},\\
&\textit{Response (R)} \triangleq \{\mathbf{I}, \mathbf{WP}, \mathbf{MP}\},
\end{aligned}$$

where $\mathbf{A} \triangleq$ *annoying*, $\mathbf{HU} \triangleq$ *hurt*, $\mathbf{F} \triangleq$ *fatal*, $\mathbf{N} \triangleq$ *not*, $\mathbf{S} \triangleq$ *somewhat*, $\mathbf{V} \triangleq$ *very*, $\mathbf{L} \triangleq$ *low*, $\mathbf{M} \triangleq$ *medium*, $\mathbf{H} \triangleq$ *high*, $\mathbf{I} \triangleq$ *ignore*, $\mathbf{WP} \triangleq$ *want to prevent* (or minimize effects), $\mathbf{MP} \triangleq$ *must prevent.*

Similarly to Section 6.4 (Problem Analysis) we can apply the simplified FLC technique considering the input variables as independent. Then the rules are reduced to 9; they are of the type (6.2)–(6.4). Denoting *potential problem* or *potential deviation* by *PD*, the selected rules are:

$$\left.\begin{array}{ll}\text{Rule 1:} & \textit{If PD is } \mathbf{A}S \textit{ then R is } \mathbf{I},\\ \text{Rule 2:} & \textit{If PD is } \mathbf{HU}S \textit{ then R is } \mathbf{WP},\\ \text{Rule 3:} & \textit{If PD is } \mathbf{F}S \textit{ then R is } \mathbf{MP},\end{array}\right\} \qquad (6.5)$$

$$\left.\begin{array}{ll}\text{Rule 4:} & \textit{If PD is } \mathbf{N}P \textit{ then R is } \mathbf{I},\\ \text{Rule 5:} & \textit{If PD is } \mathbf{S}P \textit{ then R is } \mathbf{WP},\\ \text{Rule 6:} & \textit{If PD is } \mathbf{V}P \textit{ then R is } \mathbf{MP},\end{array}\right\} \qquad (6.6)$$

$$\left.\begin{array}{ll}\text{Rule 7:} & \textit{If PD is } \mathbf{L}D \textit{ then R is } \mathbf{I},\\ \text{Rule 8:} & \textit{If PD is } \mathbf{M}D \textit{ then R is } \mathbf{WP},\\ \text{Rule 9:} & \textit{If PD is } \mathbf{H}D \textit{ then R is } \mathbf{MP}.\end{array}\right\} \qquad (6.7)$$

The first rule for instance reads: *if potential deviation is annoyingly serious then response is ignore.*

Case Study 24 *Fuzzy Logic Control for Potential Problem Analysis*

We will specify the inputs S, P, D, and the output R introduced above similarly to the variables in Case Study 23. However to avoid repetition we can define the variables under consideration using those in Case Study 23 as follows.

Urgent (U) (Fig. 6.15) is substituted by *Serious* (S),

Serious (S) (Fig. 6.16) is substituted by *Possible* (P),

Growth potential (GP) (Fig. 6.17) is substituted by *Degree* (D),

Priority of deviation (POD) (Fig. 6.18) is substituted by *Response* (R).

Also the terms of the variables U, S, GP, and POD in Case Study 23 are substituted by the terms of S, P, D, and R in this case study, correspondingly.

Then the rules (6.2)–(6.4) are substituted by the rules (6.5)–(6.7), respectively.

To make a full use of the calculations in Case Study 23 here we assume the same readings: $x_0 = 40, y_0 = 20, z_0 = 75$ on a scale [0,100] but now the base variables have different meaning; x stands for seriousness, y for possibility, and z for degree.

The firing of the rules (Fig. 6.19), the aggregation (Fig. 6.20), and the defuzzified value $\hat{v}_h \approx 47$ remain valid.

The manager, in response to the potential deviation evaluated to be 47 on a scale from 0 to 100, wants to prevent it and he/she will work to do this. The project will be hurt in case of no action.

□

6.6 Notes

1. Graham and Jones (1988) outlined financial applications where fuzzy methods were employed (some concern *if ... then* rules). They listed various computer products, suppliers, and areas of use. Cox's book (1995) contains interesting applications in business and finance; it includes two discs and provides the C^{++} code listings for programs, demonstrations, and algorithms used in the book.

2. Kepner and Tregoe wrote in 1965 (it is still of interest today):

   "The systematic analysis of potential problem is still rare. Yet it is not difficult to show that skill in analyzing and preventing or minimizing potential problems can provide the most returns for the effort and time expended by a manager. The point is so well-known that it has become an axiom: an ounce of prevention is worth a pound of cure. So few managers apply the axiom, however, that it is reasonable to assume there are major obstacles preventing them from doing so. One obstacle is that managers are generally far more concerned with correcting today's problems than with preventing or minimizing tomorrow's. This is not surprising, of course, since the major rewards in money and promotion so often go to those who show the best records of solving current problems in management, and there is rarely a direct reward for those whose foresight keeps problems from occurring. There are also other reasons why so few managers analyze and deal with potential problems. There is the common tendency to overlook the critical consequences of an action. Such consequences may be missed because they seem too disagreeable or unpalatable to face, or the consequence may be literally invisible."